

# Temporal criterion for single-frequency operation of passively $Q$ -switched lasers

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An analytical expression for the difference in buildup time between two longitudinal modes in a passively  $Q$ -switched laser resonator is developed and compared with experimental laser data. The results support the following temporal criterion for single-frequency, passively  $Q$ -switched operation: The difference in buildup time between any two longitudinal modes of the laser resonator should be comparable with or greater than the laser pulse duration to ensure single-frequency operation. © 1999 Optical Society of America

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$Q$ -switched lasers for ranging, altimetry, lidar, and other types of remote-sensing system require short pulse durations (<10 ns), TEM<sub>00</sub>-mode profiles, and high spectral purity. Ideally, single-frequency, i.e., single-longitudinal-mode, operation is desired to eliminate amplitude instabilities that are due to mode beating in direct-detection systems and is essential for the development of homodyne–heterodyne detection systems. These  $Q$ -switched lasers can be used alone or as a master oscillator in conjunction with a power amplifier.

There have been many reports of passively  $Q$ -switched all-solid-state lasers,<sup>1–4</sup> and the majority of systems have been diode laser pumped. In Ref. 1 we reported generation of 2-to-5-ns pulses at the millijoule level from a Nd:YAG laser, using a saturable absorber based on F<sub>2</sub><sup>-</sup> color centers in a LiF crystal.<sup>5,6</sup> We used a novel diode-laser transverse-pumping geometry<sup>7</sup> to obtain efficient TEM<sub>00</sub>-mode operation with a simple, two-mirror linear resonator. A window-polished slab of antireflection-coated F<sub>2</sub><sup>-</sup>:LiF material was added as the saturable absorber for  $Q$ -switched operation. The simplicity and compactness of these passive  $Q$  switches are ideal for diode-laser-pumped systems.

Sooy<sup>8</sup> showed many years ago that, compared with faster active  $Q$  switches, a relatively slowly opening, passive  $Q$  switch provides longitudinal-mode selection because of the increased pulse buildup time, which leads to more passes of the resonator and the mode-selection elements contained therein. As a result, single-frequency operation can typically be achieved with fewer or, in some cases, with no intracavity mode-selection elements at all. Limiting the use of mode-selection elements such as etalons is desirable because they can increase intracavity loss and are prone to optical damage. We show in this Letter that careful selection of resonator parameters, in particular, shorter resonator lengths, makes it possible for a simple two-mirror resonator with a passive  $Q$  switch to operate on one longitudinal mode.

According to the work of Sooy, a criterion for single-mode operation in a  $Q$ -switched laser is that the dominant mode should be at least ten times greater in peak power than any other mode. Although this factor of 10 was not substantiated in Ref. 8, it is often used to de-

termine the differential loss required between adjacent longitudinal modes, as introduced by mode-selection elements. The discrepancies between our experimental data in Ref. 1 and Sooy's model predictions show that corrections should be made. In this Letter we describe a different approach based on a dynamic analysis of the passive  $Q$ -switching process. We show that the difference in gain between adjacent longitudinal modes leads to a difference in pulse buildup times, with the highest gain mode appearing first. This represents the physical basis for a temporal criterion for single-frequency  $Q$ -switched operation: The difference in buildup time between any two longitudinal modes of the laser resonator should be comparable with or greater than the laser pulse duration to ensure single-frequency operation. We also show that in some cases, particularly in short resonators with lengths of <10 cm, there is sufficient difference in the buildup times of competing modes to ensure single-frequency operation without the use of intracavity mode-selection elements. We present a rate-equation analysis for the difference in buildup times of any pair of longitudinal modes and apply it to our previous experimental data<sup>1</sup> to demonstrate the validity of our hypothesis.

We begin with a photon rate equation in a resonator of a passively  $Q$ -switched laser (previously treated by Szabo and Stein<sup>9</sup>). We rewrite the photon rate equation to represent the  $n$ th longitudinal mode and use it to derive an expression for the modal buildup time. Assuming unsaturated amplification to the onset of absorber saturation and neglecting spontaneous emission, we find that

$$\frac{dP_n(t)}{dt} = \frac{P_n(t)}{t_l} [2\sigma_n l_a N(t) - L_n], \quad (1)$$

where  $P_n(t)$  is the intracavity photon density for the  $n$ th mode,  $\sigma_n$  is the emission cross section at the wavelength of the  $n$ th mode,  $N(t)$  is the population inversion density in the active medium of length  $l_a$ ,  $L_n$  is the round-trip loss, and  $t_l$  is the round-trip transit time.  $L_n$  includes output coupler transmission, unsaturated absorber transmission  $T_0$ , and any other insertion-loss contributions.

The  $Q$ -switched pulse begins to build up when the threshold is reached and continues until time  $t_{sn}$ , when the saturable absorber starts to bleach. We assume an initial population inversion,  $N_i$ , during this period to be constant and slightly greater than the threshold inversion,  $N_{th}$ . Under this assumption, we can integrate Eq. (1) to obtain

$$P_n(t_{sn}) = P_i \exp[(2\sigma_n l_a N_i - L_n)t_{sn}/t_l], \quad (2)$$

where  $P_i$  is the initial photon density, which is assumed to be the same for all modes, and  $N_i = (1 + \epsilon)N_{th}$  for  $\epsilon \ll 1$ . The population inversion,  $N_{th}$ , can be determined from the threshold condition

$$\exp(2\sigma_n l_a N_{th} - L_n) = 1. \quad (3)$$

Figure 1 illustrates our definition of buildup time and shows the evolution of two longitudinal modes. Under our definition of buildup time, all resonator modes would have the same photon rate at their respective buildup times. Hence we can write

$$P_n(t_{sn}) = P_m(t_{sm}), \quad (4)$$

which, by use of Eq. (2), yields

$$(2\sigma_n l_a N_i - L_n)t_{sn} = (2\sigma_m l_a N_i - L_m)t_{sm}. \quad (5)$$

The buildup time difference,  $\Delta t_s$ , between the  $n$ th and  $m$ th modes obtained from Eq. (5) is

$$\Delta t_s = t_{sn} \frac{2(\sigma_n - \sigma_m)l_a N_i - (L_n - L_m)}{2\sigma_m l_a N_i - L_m}. \quad (6)$$

Assuming a resonator with no additional mode-selection elements, and consequently  $L_n = L_m$ , we can rewrite Eq. (6) as

$$\Delta t_s = t_{sn} \frac{2(\sigma_n - \sigma_m)l_a N_i}{2\sigma_m l_a N_i - L_m}. \quad (7)$$

The remaining unknown in Eq. (7) is the buildup time,  $t_{sn}$ , at which the absorber starts to saturate. This time is obtained from Eq. (2). Then Eqs. (3) and (7) can be combined by substitution of  $(1 + \epsilon)N_{th}$  for  $N_i$  and rearranged to give

$$\Delta t_s = \ln\left(\frac{P_n}{P_i}\right) \frac{t_l}{2\sigma_n l_a N_{th}} \frac{(1 + \epsilon)}{\epsilon^2} \left(\frac{\sigma_n - \sigma_m}{\sigma_m}\right). \quad (8)$$

To use Eq. (8) we need values for  $P_n(t_{sn})$ ,  $P_i$ ,  $\epsilon$ , and the normalized gain cross-section difference or modal gain,  $(\sigma_n - \sigma_m)/\sigma_m$ .

The photon density at the onset of bleaching,  $P_n(t_{sn})$ , can be obtained from the saturation intensity of the absorber, which for  $F_2^-:LiF$  material is equal to  $5 \times 10^{18}$  photons/cm<sup>2</sup>.<sup>4</sup> Using the parameters of the Nd:YAG gain medium described in Ref. 1, we estimated  $P_n(t_{sn})$  to be equal to  $10^{19}$  photons/cm<sup>3</sup>. We evaluated  $P_i$  by considering the density of spontaneous emission into the resonator mode and found it to be  $10^8$  photons/cm<sup>3</sup>. Hence in our case  $\ln(P_n/P_i)$  was

approximately 27. Note that this value is only weakly dependent on  $P_i$  and  $P_n(t_{sn})$ .

Experimentally we were just able to observe weak, microsecond-duration laser pulses at pumping rates approximately 1.8% less than the  $Q$ -switched pumping rate, which gives  $\epsilon = 0.018$ . Note that here we consider the  $Q$ -switched pumping rate as the rate at which a single pulse is generated. (Pumping at higher rates does not necessarily result in higher pulse energy or multiple pulses until twice the pumping rate is reached, and only then is a second pulse generated.)

The term  $\sigma_n l_a N_{th}$  in Eq. (8) is equal to the total cavity loss through the threshold condition and therefore includes the effect of unsaturated absorber loss,  $T_0$ , and the output coupling. The resonator length,  $L_c$ , influences Eq. (8) through the round-trip time,  $t_l$ , and the modal-gain difference,  $(\sigma_n - \sigma_m)/\sigma_m$ . It can be shown that, assuming a Lorentzian gain profile, the modal gain difference is approximately equal to  $(L_c^2 \Delta\nu^2)^{-1}$ , where  $\Delta\nu$  is the FWHM. In our estimates of modal-gain difference,  $\Delta\nu$  is equal to  $4 \text{ cm}^{-1}$ .

We analyzed 10-cm-long and 30-cm-long laser resonators, as described in Ref. 1. The calculated buildup-time differences for adjacent modes, e.g.,  $m = n + 1$ , are shown in Figs. 2 and 3, along with the measured pulse durations,  $t_p$ , for comparison. These data clearly indicate that the buildup-time differences

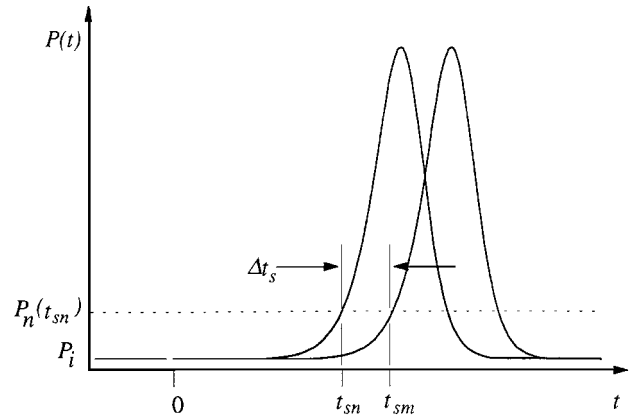


Fig. 1. Illustration of the temporal evolution of two different longitudinal modes.

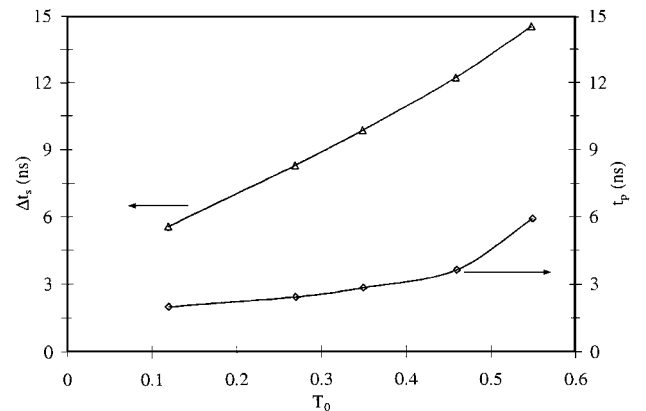


Fig. 2. Calculated pulse buildup-time difference,  $\Delta t_s$ , and measured pulse duration,  $t_p$ , as a function of unsaturated  $Q$ -switched transmission,  $T_0$ , for a 10-cm-long resonator.

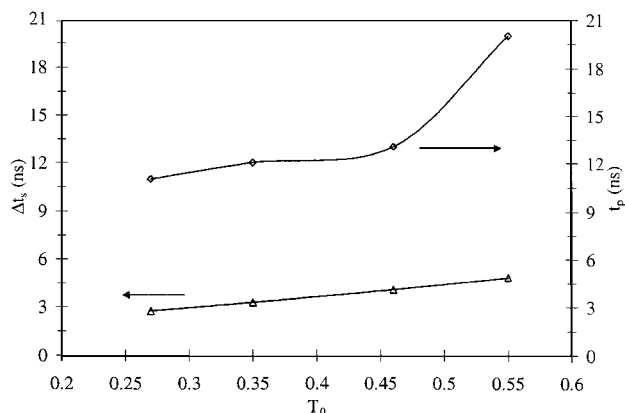


Fig. 3. Calculated pulse buildup-time difference,  $\Delta t_s$ , and measured pulse duration,  $t_p$ , as a function of unsaturated  $Q$ -switch transmission,  $T_0$ , for a 30-cm-long resonator.

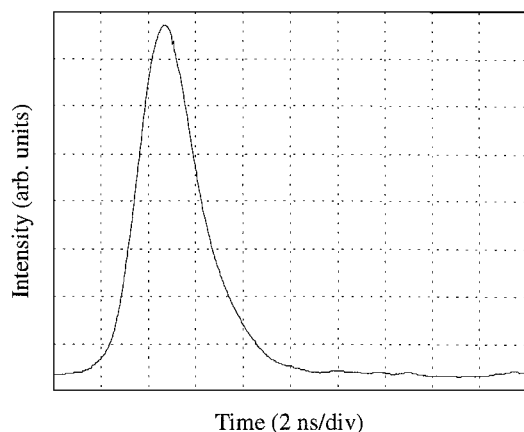


Fig. 4. Measured 2.8-ns-duration  $Q$ -switched laser pulse for the 10-cm-long resonator.

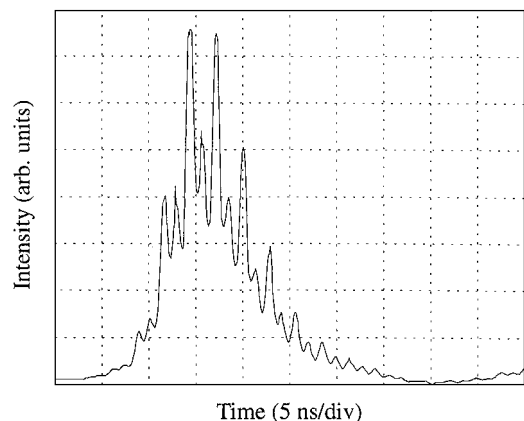


Fig. 5. Measured 11-ns-duration  $Q$ -switched laser pulse for the 30-cm-long resonator.

are greater than the pulse durations for the short resonator and much shorter than the pulse durations for the long resonator. Given this information, we would expect the long resonator to operate on several longitudinal modes, because the modes all have enough time to reach threshold before the stored energy is extracted. In the case of the short resonator the domi-

nant longitudinal mode will have extracted almost all the stored energy by the time an adjacent mode reaches threshold, and the laser pulse is expected to be single mode. To verify our calculations we compared the pulse profiles for the two resonators. Pulses from the short resonator (Fig. 4) showed no evidence of the mode beating associated with multimode operation, whereas pulses from the long resonator (Fig. 5) showed strong mode beating. Thus, as predicted, our experimental results are indicative of single-mode operation for the 10-cm resonator and of multimode operation for the 30-cm resonator.

In summary, we have demonstrated that a passively  $Q$ -switched laser can oscillate on a single longitudinal mode when the difference in buildup times between any pair of longitudinal modes is greater than the  $Q$ -switched pulse duration. This condition can be achieved without the use of additional frequency-selection elements and facilitated by use of a short resonator, a narrow-band gain medium, and a saturable absorber with a large saturation fluence. One can apply the approach developed here to the design of any passively  $Q$ -switched laser to ensure single-frequency operation. Instead of experimentally evaluating the pulse duration, one can estimate it by use of an expression such as Eq. (12) in the study reported by Degnan<sup>10</sup> and compare the result with the value of the buildup-time difference as defined in this Letter.

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